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Part 1

a) $L = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ b) $M = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ c) $N = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

$L^2 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$ $M^2 = \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$ $N^2 = \begin{pmatrix} 25 & 24 \\ 24 & 25 \end{pmatrix}$

d) $A^2 = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$

$= \begin{pmatrix} (a+1)(a+1)+a^2 & a(a+1)+a(a+1) \\ a(a+1)+a(a+1) & a^2+(a+1)(a+1) \end{pmatrix}$

$= \begin{pmatrix} 2a^2+2a+1 & 2a^2+2a \\ 2a^2+2a & 2a^2+2a+1 \end{pmatrix}$

Rewrite matrix as:

$\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

e) Xavi is right in examples a), b) and c).

f) Xavi is right when a is a real number, including zero.

Verify:

If $a = 0$,

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$= \begin{pmatrix} 0+1 & 0 \\ 0 & 0+1 \end{pmatrix}$

If $a = -1$,

$A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, $A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$= \begin{pmatrix} 0+1 & 0 \\ 0 & 0+1 \end{pmatrix}$

If $a = -2$,

$A = \begin{pmatrix} -1 & -2 \\ -2 & -1 \end{pmatrix}$, $A^2 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$
$= \begin{pmatrix} 4+1 & 4 \\ 4 & 4+1 \end{pmatrix}$

g) $LM = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 8 & 7 \\ 7 & 8 \end{pmatrix}$

h) $MN = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 18 & 17 \\ 17 & 18 \end{pmatrix}$

i) $LN = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix}$

j) $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

$= \begin{pmatrix} (a+1)(b+1) + ab & b(a+1) + a(b+1) \\ a(b+1) + b(a+1) & ab + (a+1)(b+1) \end{pmatrix}$

$= \begin{pmatrix} 2ab + a + b + 1 & 2ab + a + b \\ 2ab + a + b & 2ab + a + b + 1 \end{pmatrix}$

Rewrite as:

$\begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$ where a, b and c are real numbers, including positive, negative and zero

Messi's hypothesis is correct.

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k) $L^3 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^3 = \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$

l) $L^4 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^4 = \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$

m) $L^5 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^5 = \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$

n) If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$,

then A^3 is of the form $\begin{pmatrix} d+1 & d \\ d & d+1 \end{pmatrix}$,

A^4 is of the form $\begin{pmatrix} e+1 & e \\ e & e+1 \end{pmatrix}$,

A^5 is of the form $\begin{pmatrix} f+1 & f \\ f & f+1 \end{pmatrix}$ etc.

In A^x, where x is an odd number,

$\begin{pmatrix} d+1 & d \\ d & d+1 \end{pmatrix}$ d is an odd number

In A^y, where y is an even number,

$\begin{pmatrix} e+1 & e \\ e & e+1 \end{pmatrix}$ e is an even number

No matter what Matrix A's power is, the form will always be

$\begin{pmatrix} k+1 & k \\ k & k+1 \end{pmatrix}$ where k is a real number and non-zero.

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Part 2.

Coordinates of triangle ABC : $\begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

a) $MT = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} -1 & -1 & -3 \\ -2 & -5 & -4 \end{pmatrix}$

b) MT generates coordinates of a triangle T reflected across line $x = -y$

c) The transformation by Matrix M is reflection across the line $x = -y$

d) $M^2T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

The result of Matrix M^2T equals Matrix T because M^2 is an identity matrix. Multiplying by the identity matrix makes no difference to the original matrix here.

e) $M^3T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} -1 & -1 & -3 \\ -2 & -5 & -4 \end{pmatrix}$

The result of M^3T is the same as MT, because M and M^3 are both : $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Like Matrix MT, Matrix M^3T shows the reflected triangle across line $x = -y$.

$M = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

f) $M^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$= \frac{1}{0-(1)}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$= -1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

The answer is the same as the original Matrix. When calculating inverse, you need to change the positions of a and d, in this case, nothing changes. Then, b and c are changed from -1 to 1. However, when multiplied by $\frac{1}{det}$, which is -1, the b and c becomes negative again, like the original matrix.

g) Method 1:

find a pattern in M^xT and calculate results.

$MT = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} -1 & -1 & -3 \\ -2 & -5 & -4 \end{pmatrix}$

$M^2T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$M^3T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} -1 & -1 & -3 \\ -2 & 5 & -4 \end{pmatrix}$

$M^4T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

When x in M^xT is an even number,

$M^xT = \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

When x in M^xT is an odd number,

$M^xT = \begin{pmatrix} -1 & -1 & -3 \\ -2 & -5 & -4 \end{pmatrix}$

Since x in $M^{89998}T$ is an even number,

M^{89998} must equal $\begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

Method 2: Another method would be using the GDC. Input Matrix M, ^ 89998 and multiply Matrix T for answer.

Method 3: Multiply the Matrix M by itself 89998 times, then multiply by Matrix T.

Conclusion: Method 1 is the most appropriate because really easy manual calculation of 2x3 and 2x2 matrix are done four times and the pattern is generated. There is no need to calculate anything for $M^{89998}T$, only take answer from M^xT,

g) (continued)

, where x is an even number. You can also use the pattern for any other questions given M^xT. It does not take much time, unlike method 3 and you don't need any equipment, unlike method 2.

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Part 2. c